Modal Analysis of Parallel and Crossed Rectangular Waveguide Broadwall Couplers with Apertures of Arbitrary Shape

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Abstract— **Parallel and crossed rectangular waveguide broadwall couplers with apertures of arbitrary shape and number are analyzed rigorously by the efficient and flexible boundary-contour/mode-matching (BCMM) method. The modal S-matrix combination technique takes adequately into account both the finite wall thickness and the higher order mode interaction between the discontinuities. Examples with circular, elliptical, and cross apertures demonstrate the flexibility of the design method. The theory is verified with measurements.**

I. Introduction

Aperture coupling in the broad wall of waveguides is a well-known technique to build a useful class of directional couplers for a large variety of applications [1] - [7]. Many refined equivalent-network synthesis methods are available for deriving the necessary coupling coefficients for desired directional coupler characteristics [1] - [3], [5], [7]. The design of the practical aperture geometries, however, is still mostly based on the approximate Bethe-Cohn theory [8], [9], together with thickness correction factors [2], [3], [5], [7], [10].

For small rectangular slots in the common waveguide wall improved calculations based on the variational [11], or moment method [12] are available. A mode-matching analysis for large rectangular apertures in the broad wall of waveguides has been derived just recently [17]. For coupling apertures of arbitrary shape, no rigorous analysis has been reported so far. The increasing demand for high-quality low-cost couplers and the requirement for shorter development cycles have prompted the need for accurate field theory based CAD methods which take adequately into account the effects of large apertures, apertures of more complicated shape, finite wall thickness, and the higher-order mode interaction at all discontinuities.

This paper presents a rigorous yet efficient design method for parallel or crossed rectangular waveguide broad-wall couplers with apertures of arbitrary shape and number, Figs. 1a, b. The apertures are calculated with the flexible boundary-contour/mode-matching (BCMM) method [18], [19]. With the known mode-matching T-junction key-building block, the modal S-matrix combination takes rigorously into account both the finite wall thickness and the higher order mode interaction between the discontinuities.

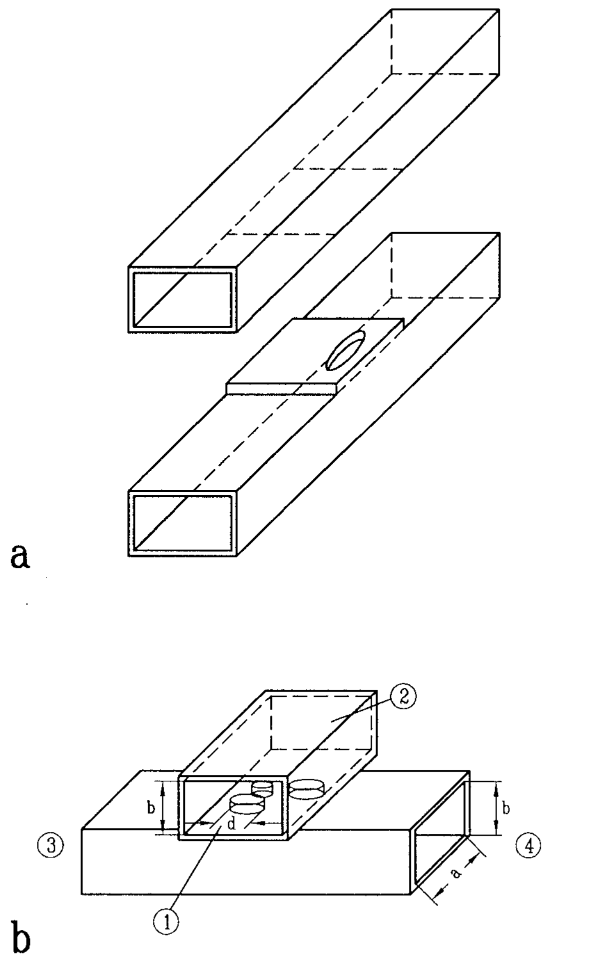

Fig. 1. Broadwall coupler with apertures of arbitrary shape and number. (a) Parallel type. (b) Crossed (Moreno) type

In contrast to e.g. recent moment method solutions for narrow slots [12], the effect of large apertures is accurately included. Moreover, the suitable modal S-

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matrix formulation for multiport structure based on the asymmetric double-step discontinuity [15] or the asymmetric transition from rectangular to smaller circular waveguide [14] allows the rigorous formulation for double-aperture structures which have turned out to be more convenient for such types of couplers [3]. Cross (MORENO) type couplers (Fig. 1b) are included by the adequate application of the T-junction building block [13] twisted by 90°.

II. THEORY

For the modal S-matrix combination of the composed structure, the required key-building block elements, the general T-junction, the general rectangular waveguide step discontinuity, and the transition from rectangular to smaller circular waveguide are already given in [13], [14], respectively. The technique for solving the problem of multiple discontinuities in a cross-section (e.g. multiple apertures), if the single discontinuity is known, is described in [15].

For the general cross-section of the coupling apertures with an analytically or numerically given contour function $r(\varphi)$, the fields are expanded in terms of the complete set of cylindrical wave functions [19]

$$T(r,\varphi) = \sum_{n=0}^{N} J_n(k_c r)\left[a_n cos(n\varphi) + b_n sin(n\varphi)\right]. \quad (1)$$

Equation (1) is then multiplied with appropriate weighting functions $cos(j\varphi), sin(j\varphi)$, and integrated along the arbitrary contour in order to satisfy the given field periodicity with respect to the angular coordinate φ. This relates the still unknown coefficients a_n, b_n in equation (1) and the corresponding Fourier coefficients α_j, β_j resulting from the contour integration, in the following manner

$$\alpha_j = \sum_{n=0}^{N}\left[\int_0^{2\pi} J_n(k_c r) a_n cos(n\varphi) cos(j\varphi) d\varphi + \int_0^{2\pi} J_n(k_c r) b_n sin(n\varphi) cos(j\varphi) d\varphi\right], \quad (2)$$

$$\beta_j = \sum_{n=0}^{N}\left[\int_0^{2\pi} J_n(k_c r) a_n cos(n\varphi) sin(j\varphi) d\varphi + \int_0^{2\pi} J_n(k_c r) b_n sin(n\varphi) sin(j\varphi) d\varphi\right]. \quad (3)$$

The requirement that the tangential electric field strength along the iris boundary contour is zero yields a homogeneous system of equations which may be written in matrix form

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \underbrace{\begin{bmatrix} [CC] & [SC] \\ [CS] & [SS] \end{bmatrix}}_{[C]} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{x} = 0\,, \quad (4)$$

where the submatrices are denoted according to the $sin-$ and $cos-$terms in equations (2) - (3). The nontrivial solutions of this system of equations (4) result from $det[C] = 0$ which yields the eigenvalues, i.e. the cut-off wave-numbers k_c, of the iris waveguide section of arbitrary shape and finite length. The eigenvalues and the eigenvectors a, b are efficiently calculated by the singular value decomposition (SVD) method.

The modal scattering matrix of the discontinuity rectangular to arbitrarily shaped waveguide at z = 0 is obtained in the usual form [13] - [19], by matching the tangential field components. This yields the following equations where e, h are the transverse vector fields, Z, Y the wave impedances and admittances, N are the normalization expressions of the corresponding waveguide sections. I and II denote the larger (rectangular) and smaller (arbitrary) waveguide section, respectively, and where z is assumed to be directed from the smaller to the larger section

$$E_t^I = \sum_i e_i^{e,I} N_i^{e,I} Z_{0,i}^{e,I}\left(a_i^{e,I} - b_i^{e,I}\right) + \sum_j e_j^{h,I} N_j^{h,I}\left(a_j^{h,I} + b_j^{h,I}\right)\Big|_{z=0}, \quad (5)$$

$$E_t^{II} = \sum_p e_p^{e,II} N_p^{e,II} Z_{0,p}^{e,II}\left(-a_p^{e,II} + b_p^{e,II}\right) + \sum_i e_q^{h,II} N_q^{h,II}\left(a_q^{h,II} + b_q^{h,II}\right)\Big|_{z=0}, \quad (6)$$

$$H_t^I = \sum_i h_i^{e,I} N_i^{e,I}\left(a_i^{e,I} + b_i^{e,I}\right) + \sum_j h_j^{h,I} N_j^{h,I} Y_{0,j}^{h,I}\left(a_j^{h,I} - b_j^{h,I}\right)\Big|_{z=0}, \quad (7)$$

$$H_t^{II} = \sum_p h_p^{e,II} N_p^{e,II}\left(a_p^{e,II} + b_p^{e,II}\right) + \sum_q h_q^{h,II} N_q^{h,II} Y_{0,q}^{h,II}\left(-a_q^{h,II} + b_q^{h,II}\right)\Big|_{z=0}. \quad (8)$$

Application of the orthogonality of the eigenfunctions and rearranging the equations yields the modal scattering matrix of the discontinuity directly in a similar manner as in [13] - [19]. For the simplification of the calculation of the coupling integrals, all double integrals are reduced to contour integrals. The modal scattering matrix of cascaded structures (e.g. iris of finite thickness) is calculated by the known generalized modal S-matrix combination technique.

For all examples calculated in this paper, only twenty cylindrical wave functions (equation (1) for the field expansion in the iris cross-section structures, and all eigenmodes up to merely 120GHz cut-off frequency (in the subsequent order of increasing cut-off frequency) are required. The results are calculated by using a standard IBM RISC6000 workstation.

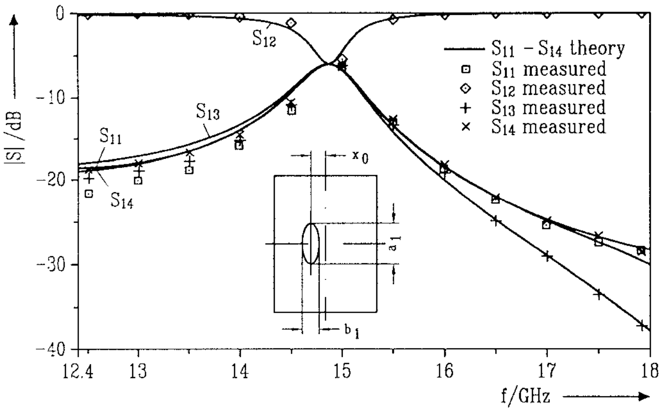

Fig. 2. Parallel R-140 (Ku-band) waveguide broad wall coupler structure with a displaced elliptical aperture. Dimensions in *mm*: $a_1 = 12, b_1 = 5, x_0 = 4.39$, thickness $t = 2.5$.

III. Results

For the verification of the analysis, first some single coupling apertures in the parallel R-140 (Ku-band) waveguide broad wall coupler structure (Fig. 1a) are investigated. Fig. 2 shows the calculated and measured scattering parameters of a displaced elliptical aperture with finite thickness. Good agreement can be stated.

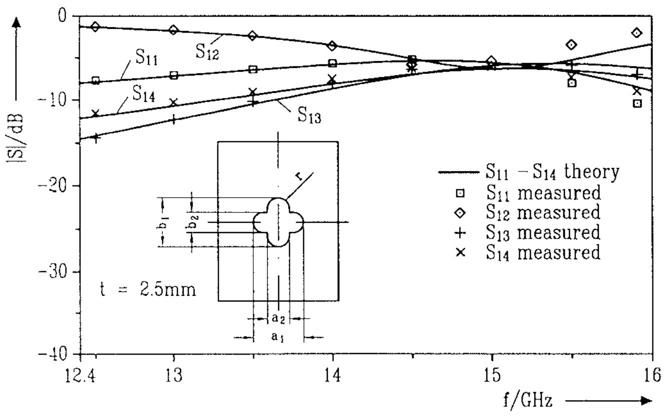

Fig. 3. Parallel R-140 (Ku-band) waveguide broad wall coupler structure with a centered cross iris aperture with rounded corners. Dimensions in *mm*: $a_1 = 7.5, b_1 = 6.6, a_2 = 5.544 (r = a_2/2), b_2 = 5.04 (r = b_2/2)$, thickness $t = 2.5$.

The second example is a cross iris with rounded corners (Fig. 3). Also this coupling structure demonstrates good agreement with measurements. The next example concerns the crossed (or Moreno) type of couplers (Fig.1b). Fig. 4 shows the calculated and measured results for a triple circular aperture coupler with dimensions according to [2]. Note that this simple coupler structure demonstrates already a rather flat coupling behavior for nearly the whole waveguide band. With the exception of the the isolation S_{14} which was beyond the dynamic range of the measuring equipment used, good agreement between theory and measurements may be stated.

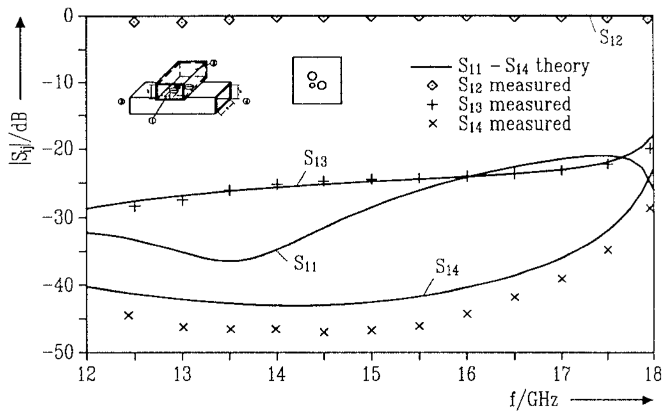

Fig. 4. Crossed R-140 (Ku-band) waveguide broad wall coupler (Moreno type) structure with a triple circular aperture. Dimensions [2], thickness $t = 2.5mm$.

A typical design example for a -30dB parallel waveguide broadwall coupler with multiple circular apertures [3] is shown in Fig. 5. The coupler shows good broadband behavior between 7.8 and 11.5 GHz by using only a double row of six apertures. The theoretical results agree well with those reported in [3].

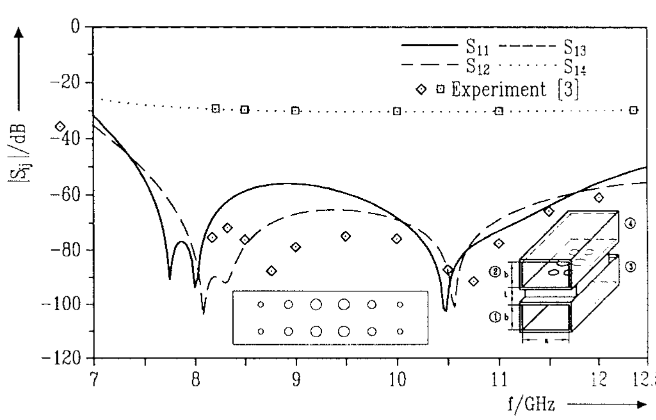

Fig. 5. Parallel R-100 (X-band) waveguide broad wall coupler with multiple circular apertures. Dimensions [3].

IV. Conclusion

A rigorous and efficient boundary-contour/mode-matching (BCMM) method for the design of parallel and crossed rectangular waveguide broad wall aperture couplers is presented which takes for the first time accurately into account the arbitrary shape of the apertures. Moreover, the effects of multiple apertures, finite wall thicknesses, and higher-order mode interactions between all discontinuities are rigorously involved. The theory is verified by good agreement with measurements.

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